SU3150 – PRINCIPLES OF GEODESY

State Plane Coordinate Systems (SPCS)

(Chapter 6)
• Plane coordinates in surveying
• Problems with plane coordinates when surveys covers a large area
• Map projections
• State Plane coordinates

Most surveys use plane coordinates for computing positions, lengths and directions

Most surveyors are familiar with computations involving plane coordinates

Why Plane Coordinates?

• Computations involving plane coordinates are simpler compared with geodetic calculations

• Plats (and maps) are made on flat medium, and generally, plane coordinates are used to plot them
Problems

- Distortions in angles, distances and areas
- Need a map projection for accurate computations

Why use map projections for plane coordinates

- Based on mathematically rigorous transformation of positions, lengths and directions
- Distortions are known (or can be computed)
- Can cover large areas
Why State Plane Coordinates?

- Unified coordinate system based on national datum
- Needed parameters are already available
- Legally recognized

Map Projections

- Distortions
- Properties
- Scale factor
- Scale error
- Elevation factor

Map projection is an orderly arrangement of two sets of lines or curves that constitute the framework of a map or a chart (graticule)
On earth, two sets of curves represent meridians and parallels

Map Projections used in SPCS

Mathematical function(s) which transform geodetic coordinates, distances and directions to corresponding quantities on a plane (grid)

Three methods of generating map projections, in general

- Geometric (perspective)
- Semi-geometric
- Mathematical

Projection is from the datum surface to a plane or a developable surface (a surface which can be developed into a plane)

Three classes of projections based on geometry of developable surface
• Azimuthal
• Conical
• Cylindrical

The projections are also classified according to the positioning of the developable surface on datum surface (ellipsoid)

• Tangent
• Secant
• Poly-superficial

Another way to classify projections is by the orientation of the developable surface (plane) or the axis (cone and cylinder) with the rotational (minor) axis of the ellipsoid

• Normal
• Transverse
• Oblique
Every map projection introduces distortions (changes) in distances, directions (angles) and areas on the datum surface i.e. these values get altered in the process of projecting from datum to projection surface.

Some distortions can be eliminated or controlled mathematically.

Three classes projections based on distortion characteristics:

- Equidistant
- Equal area
- Conformal

All three properties can be controlled by controlling the scale factor as defined below:

\[ \text{Scale Factor} = \frac{\text{Grid (Projection) Length}}{\text{Datum Length}} \]

Scale factor is a ratio of a length on projection surface to the corresponding length on the datum surface and can be larger or smaller than 1.
Scale distortion or scale error is the amount of distortion introduced into the distances, i.e. amount of deviation from a scale factor of unity

i.e. \[ S. E. = |1 - S.F.| \]

If the scale factor is very close to 1, the scale error will be small too.

The scale error of a projection increases away from the circle of tangency or intersection of the developable surface with the datum surface.

Following are the conditions needed for controlling the distortion properties of a map projection:

Equidistant \( \Rightarrow \) S.F. = 1

Equal area \( \Rightarrow \) product of S.F. in any two directions perpendicular to each other = 1

Conformal \( \Rightarrow \) S.F. in any two directions perpendicular to each other are equal
Tissot Indicatrix is a

Conformality is important in surveying because

- Preserves angular relationship (between short lines), and therefore, preserves shapes of features of limited size
- Scale factor at a point is same in every direction, i.e. independent of azimuth of line

Conformal projections result in change of

- Distances
- Azimuths

All conformal projections have a scale factor different than one, in general, and is not constant over the projection

Even though conformal projections preserve the angles, the directions of lines are changed in the process of projeting them
This is due to the fact that geodetic meridians are curved and converge toward the pole whereas grid meridians are a set of parallel, straight lines.

This angular difference between two sets of meridians, called convergence angle or mapping angle, affect azimuths of lines.

Note that, on a conformal projection, angles between intersecting lines are unchanged even though azimuth of lines may change.

Projections used in state plane coordinate systems (spcs):

- Conformal
- Uses the reference ellipsoid as datum
- Area of coverage (zone) is chosen to keep length distortions within limits acceptable to most surveyors, i.e. scale error is limited to 1:10,000
- SPCS-27 used NAD-27 datum and linear unit was foot
- SPCS-83 was computed on NAD-83 datum and uses meter as the linear unit
- Most states have enacted legislation to legalize the use of State Plane coordinates and stipulates which linear units to be used, i.e. a conversion from original coordinates is necessary
Which states use which projections?

Three conformal projections have been used in designing the SPCS by the National Geodetic Survey

- Lambert Conformal Conic
- Transverse Mercator
- Oblique Mercator

Oblique mercator projection was used for one zone in Alaska

Other two projections have been used for rest of the states, usually one projection in a given state with several zones if needed

The objective in selecting a projection and zones is to reduce the number of zones necessary to cover a state while keeping the scale error to 1:10,000 limit

Geographical shape of the state plays a part in this selection

Recall that, in every projection, the scale factor is unity along the circle(s) of tangency or intersection, and increases away from them
Lambert conformal projection is a conic projection, and is used in SPCS with a normal orientation and secant to the ellipsoid.

Circles of intersection with the ellipsoid are parallels of latitude which are called standard parallels along which the scale factor is unity.

Transverse Mercator, as it implies, has the axis of the cylinder at a right angle to the minor axis of ellipsoid and the cylinder intersects the ellipsoid along two ellipses which are perpendicular to the equator.

Scale factor along a line midway between intersecting circles (or ellipses) is the smallest and the intersecting circles are chosen in a such a way that the scale error at this midway location is within the allowed limit.

Outer limits of the projection zone are also chosen to keep the scale error within about 1:10,000 and this limits the width of a zone to approximately 156 miles.

There is no scale change in a direction parallel to intersecting circles, and therefore, there is no need to limit the length of a zone in that direction.
Above are helpful is deciding which projection is to be used for which state and how the zones should be demarcated.

Lambert projection is suitable for any state (or a zone) which has a larger east-west extent and T.M. projection is suitable for those which have larger north-south extent.

As a practical matter, zone boundaries are chosen to coincide with administrative boundaries such as county boundaries.

Projection is centered on a chosen meridian, called central meridian, and this is usually at the center of the east-west span of the zone.

Origin of the coordinate system is shifted from its true origin in order to keep all coordinates positive.

Defining constants of projections for each state/zone are found in appendix A of the state plane coordinate manual attached as appendix 2 of printed class notes.
Computations involved in SPCS are the following

- Plane coordinates from geodetic coordinates
- Geodetic coordinates from plane coordinates
- Grid scale factors necessary to convert geodetic distances to grid or vice versa
- Convergence angles and arc-to-chord corrections necessary to convert geodetic azimuth to grid azimuth and vice versa

Computation Methodology

- Manual using
  - projection equations
  - polynomial coefficients or projection tables (Lambert conformal conic only)
- Automated
Most software designed for cogo calculations and/or least squares adjustments have the ability to generate spcs for all states/zones

Reduction of Ground Distances to the Grid

Measured distances are topographic distances and they need to be first reduced to the datum before the scale factor can be applied

A suitable method should be used to reduce measured distances to the datum depending on whether the measured distances are slope distances or horizontal distances

If the measured distances are horizontal distances, then they could be multiplied by the Elevation Factor discussed earlier to reduce them to the datum

If the measured distances are slope distances, then an approach given in Chapter 5.7 of Courtse Notes could be used

Scale factor is used to reduce datum distances to grid distances

Combined factor is the product of scale factor and elevation factor and is used to convert measured horizontal distances to the grid directly
Grid Scale Factor

- Changes from point to point
- Same in every direction at a point
- Average value can be used in most projects

Even though the scale factor is different at different points, but for most practical applications s.f. computed at a point in the middle of the project area will suffice.

For more precise work, scale factor for each line need to be computed separately and if the lines are very long the average of s.f. at two ends of the line may be used.

In extreme situations, average S.F. ($k_{12}$) for a line may be computed as follows:

$$K_{12} = \frac{k_1 + 4k_m + k_2}{6}$$

where $k_m = S.F.$ at the mid point of line computed using latitude, longitude of mid-point.
Elevation Factor

- Geodetic height
- Orthometric height
- Geoid height

Elevation factor is the change in ground horizontal distance when reduced to datum surface, i.e.

\[
\text{Elevation Factor} = \frac{\text{Datum length}}{\text{Ground Hor. Length}}
\]

Elevation factor depends on the terrain in the project area and is always less than one as the topographic surface is above datum

If the project area is not very large or rugged, one elevation factor can be used for the entire project
Following table shows error introduced at different elevations

<table>
<thead>
<tr>
<th>Elevation (Ft.)</th>
<th>E.F.</th>
<th>Rel. Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>500</td>
<td>0.99997611</td>
<td>1: 41,852</td>
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<tr>
<td>1000</td>
<td>0.99995221</td>
<td>1: 20,926</td>
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<tr>
<td>1500</td>
<td>0.99992832</td>
<td>1: 13,950</td>
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<tr>
<td>2000</td>
<td>0.99990443</td>
<td>1: 10,464</td>
</tr>
<tr>
<td>2500</td>
<td>0.99988054</td>
<td>1:   8,371</td>
</tr>
</tbody>
</table>

In areas where the scale factor is greater than one, elevation factor reduces the scale error and when the scale factor is smaller than one, it increases the scale error.

Elevation factor should be calculated using geodetic heights (ellipsoidal heights) not orthometric heights (elevations referenced to geoid).

Ellipsoidal heights are directly computed from GPS observations, or can be computed from orthometric heights by adding geoid height (geoid undulation).

The average geoid undulation in the contiguous u.s. is about 30 m. (100 feet) and is negative.
If geoid height is disregarded, i.e. if benchmark elevations are used instead of geodetic heights in converting topographic lengths to geodetic lengths, the error introduced is approximately 1:208,000 in conterminous U.S.

Summary

- Ground horizontal lengths are converted to datum lengths by multiplying by elevation factor.
- Elevation factor is computed using the average geodetic height of the project area.
- In less accurate work, benchmark elevations (orthometric heights) can be used instead of geodetic heights.
- Geodetic lengths are converted to grid lengths by applying grid scale factor.
- Although grid scale factor changes from point to point, an average value can be used in a project.
- Elevation Factor and Grid Scale Factor can be combined into a Combined Factor.
- If ground lengths are used instead of grid lengths to compute State Plane coordinates, an error as much as 1:10,000 may be introduced by this cause alone.
- Final error in coordinates may be larger as there are also measurement errors.
Reduction of Measured Azimuths to the Grid

- Astronomic azimuth
- Geodetic azimuth
- Grid azimuth
- Convergence angle
- Arc-to-chord correction

SPCS computation formulas have been developed for conversion of geodetic azimuths to grid azimuths.

If the orientation of the survey is by astronomic azimuth observations, they need to be first converted to geodetic azimuths by applying Laplace correction.

Deflection of vertical values needed for computing Laplace correction can be generated using software DEFLEC available from NGS.

Difference between geodetic azimuth and grid azimuth is due to

- Difference of geodetic and grid meridians (convergence angle)
• Curvature of the projected survey line (arc to chord correction)

\[ \text{Grid Az.} = \text{Geod. Az.} - \text{Converg. Angle} + \text{Arc to Chord Correction} \]

\[ t = \alpha - \gamma + \delta \]

Convergence Angle

• Can be positive or negative

• Zero at central meridian and increases in magnitude away from C.M.

In Lambert's Conic projection, projected geodetic meridians (not grid meridians) too are straight lines, and therefore, the size of the convergence angle only depends of the longitude change from the central meridian to the point in question.
In T.M. projection, projected geodetic meridians are curves, and the size of the convergence angle depends not only on the longitude change but also on the latitude of the point.

Projected survey lines too are curved on both Lambert and T.M. projections, and this requires another correction, called *arc-to-chord (t−T)* correction, which is the angular difference between the projected survey line (arc) and straight grid line (chord).

Arc to chord correction is usually insignificant for short lines (e.g. less than 5 miles). Magnitude of arc to chord correction depends on:

- Length of line
- Distance from central axis of projection

Algebraic sign of arc to chord correction depends on:

- Azimuth of line
- Which side of the central axis

Axis of the projection is the central parallel (Lambert) or central meridian (T.M.) where the S.F. is smallest.
Steps in Computing State Plane Coordinates

1. Determine State Plane coordinates, grid scale factor, and convergence angle at the starting point. This usually is a national control point and above values are available in Data Sheets.

2. Reduce all measurements to the grid
   
   1a Convert measured ground lengths to grid lengths by applying appropriate conversion factors.
   1b Convert measured azimuth to grid azimuths by applying convergence angle, and if necessary, the arc-to-chord correction

3. Compute coordinates using standard plane coordinate computation methodology

4. Check for any closing errors, if applicable
Traverse Computations with State Plane Coordinates

Follow 10 steps given on p.54 of spcs manual

Step 5:

In low order surveys, ground horizontal distance may be used but not recommended

Distances should be reduced to grid if any of the following are true

- High precision is needed to be maintained in the survey
- Lines are very long, e.g. 5 miles or longer
- Project is at a higher elevation

Step 8:
All measured angles are very closely equal to the grid angles (conformality)

For angles measured between very long lines, arc-to-chord \((t-T)\) may be needed correction to angle = correction to F.S - correction to B.S
Azimuths of traverse lines may be now computed as in any traverse

Final azimuth may be compared with a closing grid azimuth

State plane coordinates are computed with grid azimuths and grid lengths

Table on next page shows coordinates computed before and after applying \((t-T)\) correction
<table>
<thead>
<tr>
<th>POINT</th>
<th>NORTHING</th>
<th>EASTING</th>
</tr>
</thead>
<tbody>
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</tr>
<tr>
<td></td>
<td>58,950.539</td>
<td>673,992.969</td>
</tr>
</tbody>
</table>
Universal Transverse Mercator (UTM) Projection

Developed by the U.S. military for world-wide coverage

Initially used clarke 1866 ellipsoid as datum and covered a latitude span of 80° S to 80° N

Currently uses GRS-80 in North America and the coverage only from 0° to 80° N latitude

Each zone covers a longitude span of 6° and hence there are 60 zones covering the entire globe

Meter is the linear unit

UTM is not suitable as a survey grid as scale change is too large in most areas and cannot be disregarded

It is widely used as a coordinate control for GIS data layers as each zone has a larger geographic coverage