

LS443 INTRODUCTION TO GEODESY

COURSE GUIDE

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INDRAJITH D. WIJAYRATNE
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**SCHOOL OF TECHNOLOGY
MICHIGAN TECHNOLOGICAL UNIVERSITY
HOUGHTON, MICHIGAN 49931-1295**

LS443 - INTRODUCTION TO GEODESY

Credit Hours: 3(0-3-0)

Course

Description: Geometrical and physical properties of the earth. Geodetic datums and coordinate systems. Conformal mapping and State Plane coordinate systems. Gravity field of the earth and its effect on surveying and leveling measurements. Prerequisites: LS352 and MA150 or MA160.

Textbook: There is no text for this course. A set of notes developed by the instructor is available at the university bookstore. Additional references, including the following publications, can be found in the university library.

References:

1. "Basic Geodesy", by J.R. Smith
2. "Introduction to Geodesy" by Clair E. Ewing
3. Geodesy: Concepts by Krakiwisky, Edward and

Grading:	Midterm	25%
	Final Exam	36%
	4 Assignments	24%
	Quizzes	15%

Note: Quiz with the lowest grade will be dropped. Class attendance and completion of all assignments will be considered in assigning a completion grade.

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TR 9:00-10:00

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Goals/Objectives: The objective of this course is to expose the students to the basic concepts of geodesy and geodetic surveying. Field and computational techniques frequently encountered by professional surveyors will be discussed. The U.S. State Plane coordinate system will be studied along with sample calculations using computer software. Geodetic computational techniques are becoming more important with the advent of high caliber instruments and newer technologies such as GPS. An introduction to the Universal Transverse Mercator coordinates will also be given.

The students are expected to have a thorough understanding of standard surveying techniques, coordinate systems and computations. Knowledge of 2-D and 3-D geometrical concepts and relevant mathematical treatments are essential for this course, and the ability to visualize objects in 3-D space is an advantage.

Course Outline

Week	Topic	Chapter
1	Introduction, History and Development of Geodesy, Geodetic Datums and Control Networks	1, App 1
2	Understanding the Geometry of Reference Ellipsoid Reference Coordinate Systems, Coordinate or datum conversion	2.1, 2.2
3	Lines (curves) on the Ellipsoid, Normal sections and geodetic lines, Radii of curvature and calculation of lengths of lines	2.3, 2.4
4	Direction of lines on the ellipsoid, Convergence of Meridians, Deflection of vertical and Laplace correction	2.5, 2.6
5	Reduction of measured distances to Reference datum geodetic coordinate computations	2.8 2.9
6	Map Projections and Plane Coordinate systems Review and Discussion, Exam-I	Chap 1 of Appendix 2
7	State Plane Coordinates, Definition and Implementation, Coordinate Conversion in State Plane Coordinates	Chap 2, App 2 Chap 3, App 2
8	Reduction of Lengths and Azimuths, UTM Coordinates	Chap 4, App 2
9	Gravity Field of the Earth, Theoretical Considerations,	3.1
10	Measurement of Gravity, Determination of Geoid, Orthometric Heights	3.2, 3.3

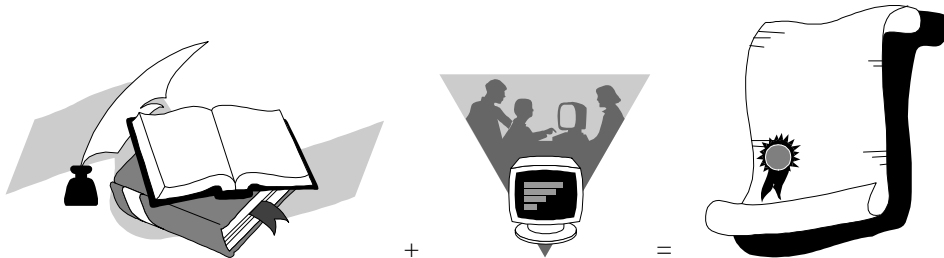
Guidelines for turning in assignments

1. Every assignment is due on the due date indicated. Late assignments will only be accepted from those who have made prior arrangements. This will only be for exceptional circumstances.
2. All assignments should be submitted on 8 1/2 x 11 sheets of paper and should be legible. Pages should be numbered and stapled together. No loose sheets will be accepted. Write your name, assignment number and the course # clearly on the first page.
3. You are strongly encouraged to work the problems in groups, but each individual should have a clear understanding of the problem and its solution. For this reason, each individual must write up his or her solution. You are also encouraged to ask questions about assignments in class.
4. Solutions to problems should be presented clearly and in a logical order in sufficient detail to enable one to follow it easily . **All work and formulas used must be clearly shown and values obtained using computer programs must be shown with the solution and not merely refer to computer printouts.** The reasoning or logic used in obtaining the solution must be clearly indicated.

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WHAT IS GEODESY?

'SCIENCE WHICH DETERMINES THE SIZE AND SHAPE OF THE EARTH, THE PRECISE POSITIONS AND ELEVATIONS OF POINTS, LENGTHS AND DIRECTIONS OF LINES ON THE EARTH'S SURFACE, AND THE TERRESTRIAL GRAVITY AND ITS VARIATIONS'

FUNCTIONS OF GEODESY

- ESTABLISHING REFERENCE DATUMS AND COORDINATE SYSTEMS FOR THE DEFINITION OF
 - HORIZONTAL POSITIONS OF POINTS
 - DISTANCES AND DIRECTIONS BETWEEN POINTS
 - ELEVATIONS OF POINTS
- MATHEMATICAL PROJECTIONS NECESSARY TO DEPICT THE DATUM SURFACE ON A MAP
- DETERMINATION OF GEOPHYSICAL PROPERTIES SUCH AS THE GRAVITY FIELD ON OR NEAR THE SURFACE OF THE EARTH, GEOID (MEAN SEA LEVEL) AND DEFLECTION OF VERTICAL (PLUMBLINE)
- STUDY AND MONITOR THE GEODYNAMICS PHENOMENA SUCH AS OCEAN AND EARTH TIDES, CRUSTAL (TECTONIC) MOVEMENTS, POLAR MOTION, VARIATIONS IN EARTH ROTATION AND GRAVITY FIELD

OBSERVATION TECHNIQUES OF GEODESY

- ASTRONOMICAL
- TERRESTRIAL
- SPACE

ASTRONOMIC OBSERVATIONS

- LATITUDE, LONGITUDE
- AZIMUTH
- VERY LONG BASELINE INTERFEROMETRY (VLBI)
- OBSERVATIONS NECESSARY TO MONITOR POLAR MOTION, PRECESSION, NUTATION, ETC.

TERRESTRIAL OBSERVATIONS

- ARC MEASUREMENTS (HISTORIC)
- TRIANGULATION, TRILATERATION, TRAVERSING
- LEVELING
- ZENITH OR VERTICAL ANGLES
- GRAVITY

SPACE BASED OBSERVATIONS

- LUNAR LASER RANGING
- SATELLITE LASER RANGING
- SATELLITE POSITIONING
- SATELLITE ALTIMETRY

APPLICATIONS OF GEODESY

- SURVEYING AND MAPPING
- DEFENSE
- GEOPHYSICAL EXPLORATIONS
- SPACE EXPLORATIONS
- COMMUNICATION, NAVIGATION, ETC.

GEODESY IS THE COMMON LINK CONNECTING

- SURVEYING
- PHOTOGRAMMETRY
- CARTOGRAPHY
- GEODYNAMICS
- GEOPHYSICS
- PHYSICS

BRANCHES OF GEODESY

- GEOMETRIC GEODESY
- GRAVIMETRIC (PHYSICAL) GEODESY
- SATELLITE GEODESY

GEOMETRIC GEODESY DEALS WITH THE SHAPE AND SIZE OF EARTH, DISTANCE AND DIRECTION OF LINES ON EARTH, REFERENCE DATUMS, AND COORDINATE SYSTEMS

GRAVIMETRIC OR PHYSICAL GEODESY IS THE SCIENCE THAT STUDIES GEOPHYSICAL AND GEODYNAMIC PROPERTIES OF EARTH, AND INCLUDES EARTH GRAVITY FIELD AND ATTRACTIONS OF SUN, MOON AND PLANETS

STUDY OF SATELLITE ORBITS, MOTION, PERTURBATIONS, AND SATELLITE BASED POSITIONING FALLS UNDER SATELLITE GEODESY

WHY SHOULD A SURVEYOR STUDY GEODESY?

- GEODETIC CONTROL SURVEYS
 - UNDERSTANDING GEODETIC DATUMS AND COORDINATE SYSTEMS, E.G. NAD27, NAVD88, WGS84
 - DIFFERENCE BETWEEN GEODETIC AND ASTRONOMIC COORDINATES
 - DIFFERENCE BETWEEN GEODETIC AND ASTRONOMIC AZIMUTH
 - AZIMUTH CHANGE DUE TO CONVERGENCE OF MERIDIANS
 - LENGTHS OF LINES ON THE DATUM SURFACE
 - REDUCTION OF MEASURED LINES TO DATUM SURFACE

- GEODETIC LEVELING

- DATUM (GEOID)
- ORTHOMETRIC HEIGHT (MEAN SEA LEVEL ELEVATION)
- EFFECT OF GRAVITY

- GPS SURVEYS
 - SATELLITE DATUM (S)
 - GPS DERIVED COORDINATES AND BASELINES
 - GPS DERIVED ELEVATIONS

- STATE PLANE COORDINATES
 - WHY IS IT NEEDED?
 - DEFINITION AND IMPLEMENTATION
 - RELEVANT COMPUTATIONS

A KNOWLEDGE OF GEODETIC PRINCIPLES ARE NEEDED IN ANY SURVEY WHICH

- COVERS A VERY LARGE AREA OR A DISTANCE
- HAS TO MEET VERY HIGH ACCURACY STANDARDS
- USES SPECIALIZED TECHNIQUES SUCH AS GPS

HISTORY OF GEODESY

- FLAT EARTH CONCEPT (HOMER, 9TH CENTURY B.C.)
- IDEA OF SPHERICAL EARTH
- (PYTHAGORAS, ARISTOTEL, NEWTON)
- FIRST ATTEMPT AT MEASUREMENTS BY ERATOSTHENES (220 B.C.) IN EGYPT

- LATER ATTEMPTS BY
 - POSEIDONIUS (135-150 B.C.)
 - I-HSING (CHINA, 8TH CENTURY A.D.)
 - CALIPH AL-MAMUN (ARAB, 820 A.D.)

- METHOD OF TRIANGULATION BY DUTCH MATHEMATICIAN SNELLIUS (1615-1620)

- CONTROVERSY OVER OBLATE SPHEROID VS. PROLATE SPHEROID (PUMPKIN VS. EGG SHAPE)

GEODETIC CONTROL SURVEYS

- WHY ARE THEY NECESSARY?
- METHODS USED
- ACCURACY STANDARDS
- CONTROL NETWORKS
- NATIONAL GEODETIC CONTROL NETS

GEODETIC CONTROL NETS WITH PERMANENT MONUMENTS INDICATING CONTROL POINTS HAVE ALWAYS BEEN A PART OF SURVEYING PRACTICE

THEY MAY BE NATIONAL CONTROL NETS, USUALLY ESTABLISHED AND MAINTAINED BY A GOVERNMENT AGENCY, OR LOCAL NETS, ESTABLISHED AND MAINTAINED BY LOCAL AUTHORITIES OR PRIVATE COMPANIES

THESE CONTROL POINTS CAN BE USED TO TIE SURVEYS THAT NEED

- HIGH ACCURACY
- COORDINATES ON THE NATIONAL OR A PREVIOUSLY ESTABLISHED DATUM

THEY HELP TIE ISOLATED AND DISJOINTED SURVEYS TOGETHER

DATA FOR GEOGRAPHIC INFORMATION SYSTEMS NEED TO BE ON A COMMON COORDINATE SYSTEM AND CONTROL NETS ARE USEFUL IN THIS ENDEAVOR

SURVEYS THAT NEED STATE PLANE COORDINATES NEED TO HAVE TIES TO THE NATIONAL CONTROL NET

HORIZONTAL CONTROL NET

- FIVE ORDERS OF ACCURACY
- PERMANENT MONUMENTS ON GROUND
- POSITION IS DEFINED ON THE DATUM
- APPROXIMATE ELEVATIONS ONLY KNOWN
- AZIMUTH MARKS WHEN CONTROL POINTS ARE NOT INTERVISIBLE

VERTICAL CONTROL NET

- THREE ORDERS OF ACCURACY
- PERMANENT BENCHMARKS
- APPROXIMATE HORIZONTAL POSITION ONLY KNOWN

NATIONAL SPATIAL REFERENCE SYSTEM

- ALL GEODETIC CONTROL DATA CONTAINED IN NGS DATABASE
- DATA FROM NGS AND THOSE SUBMITTED BY OTHER PUBLIC AND PRIVATE ORGANIZATIONS
- DATA IN BLUE BOOK FORMAT

DATA INCLUDES

- ALL HORIZONTAL AND VERTICAL CONTROL DATA
- PRECISE ASTRONOMIC DATA
- GRAVITY DATA
- SATELLITE DATA

DATUMS AND COORDINATE SYSTEMS

- WHAT IS A DATUM?
- WHY IS IT NECESSARY?
- HORIZONTAL AND VERTICAL
- REGIONAL OR GLOBAL
- NORTH AMERICAN HORIZONTAL AND VERTICAL DATUMS

DATUM IS A REFERENCE FOR DEFINING, COMPUTING, AND RELATING POSITIONS OF POINTS ON THE SURFACE OF EARTH, INCLUDING THEIR ELEVATIONS

IT USES A GEOMETRICAL FIGURE THAT ENABLES DEFINITION AND COMPUTATION OF POSITIONS OF POINTS USING KNOWN MATHEMATICAL RELATIONSHIPS

REFERENCE ELLIPSOID

- AN IMAGINARY SURFACE
- MATHEMATICALLY DEFINED
- BEST GEOMETRIC REPRESENTATION (BEST FITTING) OF THE GEOID
- USED AS A DATUM FOR HORIZONTAL POSITIONS

GEOID

THE GEOID IS THE EQUIPOTENTIAL SURFACE OF THE EARTH'S ATTRACTION AND ROTATION WHICH, ON THE AVERAGE, COINCIDES WITH MEAN SEA LEVEL IN THE OPEN OCEAN

GEOID HAS FOLLOWING PROPERTIES

- A REAL (PHYSICAL) SURFACE
- AN EQUI-POTENTIAL SURFACE OF GRAVITY
- GRAVITY VECTOR (VERTICAL) AT ANY POINT IS PERPENDICULAR TO THE SURFACE
- CLOSELY AGREES WITH MEAN OCEAN SURFACE
- DETERMINED BY OBSERVATIONS
- USED AS A DATUM FOR ELEVATIONS
- CANNOT BE REPRESENTED MATHEMATICALLY, AND THEREFORE, CANNOT BE USED AS A HORIZONTAL DATUM

NORTH AMERICAN GEODETIC DATUMS

- HORIZONTAL AND VERTICAL DATUMS SEPARATELY DEFINED
- NAD-27 AND NAD-83
- NGVD-29 AND NAVD-88

NORTH AMERICAN DATUM OF 1927 USED CLARK 1866 ELLIPSOID AS THE REFERENCE ELLIPSOID

ORIGIN WAS AT MEADES RANCH, KANSAS AND THE ELLIPSOID WAS BEST FITTING ONLY IN THE NORTH AMERICAN REGION, I.E. IT WAS A REGIONAL DATUM

NORTH AMERICAN DATUM OF 1983 REPLACED NAD-27 AND GEODETIC REFERENCE SYSTEM 1980 ELLIPSOID WAS ADOPTED AS THE REFERENCE ELLIPSOID

THIS IS A GLOBAL DATUM, I.E. BEST FITTING
WORLD-WIDE, WITH THE CENTER OF
ELLIPSOID COINCIDING WITH THE MASS
CENTER OF EARTH

THE ORIGIN OF THE COORDINATE SYSTEM IS
EQUATOR AND GREENWICH MERIDIAN FOR
LATITUDES AND LONGITUDES, RESPECTIVELY

ALONG WITH THE REDEFINITION OF NORTH
AMERICAN HORIZONTAL DATUM, ENTIRE
HORIZONTAL GEODETIC NET WAS ADJUSTED
WITH A LARGE AMOUNT OF NEW
OBSERVATIONS

REASONS FOR READJUSTMENT (WEAKNESSES IN NAD-27)

- OLD ADJUSTMENT DID NOT INCLUDE ATLANTIC SEABOARD CONTROL
- LENGTH CONTROL WAS SIGNIFICANTLY DEFICIENT
- A NUMBER OF AZIMUTHS OF INFERIOR QUALITY
- CONTROL IN ALASKA CONNECTED ONLY BY A SINGLE ARC OF TRIANGULATION
- TECTONIC MOVEMENTS AS GREAT AS 5 CM/YEAR FOUND IN SOME AREAS IN THE CONTINENTAL U.S.

BENEFITS OF USING OR CHANGING TO NAD-83

- BETTER CLOSE IN SURVEYS BETWEEN CONTROL STATIONS
- CONSISTENT WITH GPS SURVEYS
- ADDITIONAL CONTROL POINTS AVAILABLE FOR USE
- GEODETIC AND STATE PLANE COORDINATES READILY AVAILABLE FROM NGS
- FEDERAL SURVEYING AND MAPPING AGENCIES ONLY PUBLISH NAD-83 DATA
- SURVEYS PERFORMED FOR FEDERAL AND STATE AGENCIES REQUIRE USE OF NAD-83
- WGS-84 AND NAD-83 ARE THE SAME

NORTH AMERICAN GEODETIC VERTICAL DATUM OF 1929 WAS REPLACED BY NORTH AMERICAN VERTICAL DATUM OF 1988

NGVD 1929

- CONNECTED TO CANADIAN GEODETIC LEVELING NET
- 21 U.S. AND 5 CANADIAN REFERENCE TIDE STATIONS
- MORE THAN 100,000 KM OF U.S. AND CANADIAN LEVEL LINES

WEAKNESSES

- LEVELING MEASUREMENTS WERE NOT OF GOOD QUALITY
- VARIATIONS IN LOCAL MEAN SEA LEVEL AT DIFFERENT TIDE STATIONS
- NOT CONSISTENT WITH INTERNATIONAL GREAT LAKES DATUM

BENEFITS OF NAVD 1988

- A SINGLE DATUM FOR NORTH AMERICA
- IMPROVED BENCH MARK ELEVATIONS
- A SINGLE DATABASE WITH EASY ACCESS BY THE USER FOR CRUSTAL MOTION STUDIES, HEIGHTS, ELEVATION DIFFERENCES, ETC.
- IMPROVED GEOID MODEL USING GPS AND NAVD 88 DATA
- SMOOTH TRANSITION FOR ORTHOMETRIC HEIGHT DIFFERENCES DERIVED BY GRAVITY/CONVENTIONAL LEVELING AND GPS

ELLIPSOID AS A REFERENCE DATUM

- DEFINITION
- PROPERTIES

RECALL THAT THE ELLIPSOID WAS DESCRIBED AS

- AN IMAGINARY SURFACE
- MATHEMATICALLY DEFINED
- BEST GEOMETRIC REPRESENTATION (BEST FITTING) OF THE GEOID
- USED AS A DATUM FOR HORIZONTAL POSITIONS

THE ELLIPSOID USED FOR THE REFERENCE DATUM IS AN ELLIPSOID OF REVOLUTION OR ROTATIONAL ELLIPSOID, KNOWN AS AN OBLATE SPHEROID IN EUROPE

IT IS GENERATED BY ROTATING A PLANE ELLIPSE AROUND ITS MINOR AXIS

THE SIZE AND SHAPE, I.E. MAJOR AXIS AND ECCENTRICITY OF THE ORIGINAL ELLIPSE, ARE CHOSEN TO HAVE MINIMUM DEVIATIONS FROM THE GEOIDAL SURFACE, I.E. BEST FITTING

MINOR AXIS OF THE ELLIPSOID COINCIDES OR PARALLEL WITH THE MEAN ROTATIONAL AXIS OF THE EARTH AND EQUATORIAL PLANE OF THE ELLIPSOID COINCIDES OR PARALLEL WITH THE MEAN EQUATORIAL PLANE OF THE EARTH

THREE DIMENSIONAL POSITIONS OF POINTS CAN BE DEFINED IF NECESSARY, BUT VERTICAL COMPONENTS REFERENCED TO THE ELLIPSOID HAVE NO PRACTICAL USE AS THEY DO NOT CORRESPOND TO PHYSICAL PROPERTIES OF EARTH

GEODETIC COORDINATES

- LATITUDE, LONGITUDE
- SPACE RECTANGULAR
- DATUM/COORDINATE TRANSFORMATION

LATITUDES AND LONGITUDES ARE CURVILINEAR COORDINATES, I.E. ANGLES SUBTENDED BY ARCS ON THE ELLIPSOID

EQUATOR IS THE ORIGIN FOR LATITUDES AND GREENWICH REFERENCE MERIDIAN IS THE ORIGIN FOR LONGITUDES

LATITUDES ARE DEFINED WITH RESPECT TO THE NORMAL TO THE ELLIPSOID, AND THE NORMAL DOES NOT PASS THROUGH THE CENTER OF ELLIPSOID UNLIKE THE RADIUS OF A SPHERE

LATITUDES ARE EITHER NORTH OR SOUTH
AND CAN ONLY BE BETWEEN 0° AND 90°

LONGITUDES ARE GENERALLY GIVEN AS
EAST OR WEST (OF GREENWICH) AND CAN BE
BETWEEN 0° AND 180°

ALL LONGITUDES IN THE CONTIGUOUS U.S.
ARE WEST LONGITUDES

POSITION OF A POINT ON THE ELLIPSOID CAN
ALSO BE DENOTED BY 3-D, RECTANGULAR
CARTESIAN COORDINATES

THIS IS USEFUL IN SATELLITE BASED
POSITIONING AND SOME GEODETIC
COMPUTATIONS

CURVES ON THE ELLIPSOID

- NORMAL SECTIONS
- GEODESIC
- PARALLELS OF LATITUDE
- RADII OF CURVATURE

NORMAL SECTION IS SIMILAR TO A GREAT CIRCLE ON A SPHERE, AND IS THE OBSERVED SURVEY LINE IF THE SURVEY IS DONE ON THE ELLIPSOID AND THE VERTICAL AXIS OF THE THEODOLITE COINCIDES WITH THE NORMAL

SINCE NORMALS AT TWO DISTINCT POINTS ON THE ELLIPSOID ARE NOT PARALLEL, IN GENERAL, OBSERVATIONS MADE FROM TWO POINTS CREATE TWO DISTINCT NORMAL SECTIONS (RECIPROCAL NORMAL SECTIONS) THIS DIFFERENCE IS HARDLY NOTICEABLE IN ANY SURVEY, AND IS GENERALLY NEGLECTED, UNLESS ULTRA HIGH PRECISION IS REQUIRED

GEODESIC OR GEODETIC LINE IS THE COMMON LINE BETWEEN TWO POINTS AND IS THE SHORTEST DISTANCE BETWEEN TWO POINTS ON THE ELLIPSOID EVEN THOUGH THE DIFFERENCE IN LENGTH BETWEEN THE GEODESIC AND NORMAL SECTION IS INSIGNIFICANT FOR ANY LINE ENCOUNTERED IN SURVEYING

IF TWO POINTS ARE ON THE SAME MERIDIAN OR ON THE PARALLEL OF LATITUDE, I.E. BOTH POINTS HAVE EITHER THE SAME LONGITUDE OR SAME LATITUDE, THEN THE TWO NORMALS ARE COPLANER, AND THE DIRECT NORMAL SECTION, INVERSE NORMAL SECTION AND GEODESIC ARE ALL ONE LINE NORTH-SOUTH NORMAL SECTION (MERIDIAN SECTION) AND EAST-WEST NORMAL SECTION (PRIME VERTICAL SECTION) AT ANY POINT ARE THE PRINCIPAL NORMAL SECTIONS AT THAT POINT

PARALLELS OF LATITUDE (EXCEPTING THE EQUATOR) ARE NOT NORMAL SECTIONS, AND CANNOT BE SIGHTED OR RUN DIRECTLY BY ANY KNOWN SURVEY TECHNIQUE

SPECIAL SURVEY TECHNIQUES SUCH AS SECANT METHOD OR TANGENT METHOD WERE USED IN U.S. RECTANGULAR SURVEYS (P. 53-55, BLM MANUAL-1973) TO ESTABLISH PARALLELS OF LATITUDE

ALL OF THE LINES (CURVES) DESCRIBED ABOVE, EXCEPT PARALLELS OF LATITUDE, HAVE DIFFERENT CURVATURE AT DIFFERENT POINTS ON THE ELLIPSOID

CURVATURE OF A CURVE IS ALSO DEPENDENT ON THE AZIMUTH OF THE LINE, BUT DOES NOT DEPEND ON THE LONGITUDE

OF THE POINT BECAUSE OF ROTATIONAL
ELLIPSOID

THE RADIUS OF CURVATURE IS UNIQUE AT
EVERY POINT ON A CURVE EXCEPT ON A
PARALLEL OF LATITUDE WHICH IS A CIRCLE

RADIUS OF CURVATURE IS NEEDED FOR
COMPUTATION OF LENGTHS OF LINES ON
THE ELLIPSOID

RADIUS OF CURVATURE OF ANY LINE IS
COMPUTED IN TERMS OF RADII OF
CURVATURE OF PRINCIPAL NORMAL
SECTIONS

LENGTH OF LINES (ARCS) ON THE SURFACE
OF ELLIPSOID

- ALONG A MERIDIAN
- ALONG A PRIME VERTICAL
- ALONG A PARALLEL OF LATITUDE
- ALONG A LINE IN AN ARBITRARY DIRECTION

SINCE THE CURVATURE, AND HENCE THE RADIUS OF CURVATURE, CHANGES ALONG A CURVE IN GENERAL, LENGTHS CAN ONLY BE COMPUTED BY A PROCESS OF INTEGRATION

NUMERICAL PROCEDURES HAVE BEEN DEVELOPED TO COMPUTE ARC LENGTHS BUT THESE CAN ONLY BE DONE ON A COMPUTER WITH SOFTWARE

MOST LINES ENCOUNTERED IN SURVEYING ARE CONSIDERED RELATIVELY SHORT (LESS THAN 20 MILES), AND THE COMPUTATION CAN BE SIMPLIFIED BY ASSUMING THESE ARCS TO BE CIRCULAR

A SUITABLE RADIUS FOR THE CIRCULAR ARC IS CHOSEN DEPENDING ON THE NEEDED ACCURACY

GAUSSIAN MEAN RADIUS OF CURVATURE, COMPUTED USING AN AVERAGE LATITUDE FOR THE PROJECT AREA, IS COMMONLY USED UNLESS VERY HIGH ACCURACY NEEDED IN WHICH CASE RADIUS OF CURVATURE COMPUTED FOR EACH LINE IS USED

LENGTHS OF A MERIDIAN ARC CAN BE COMPUTED BY USING THE MERIDIAN RADIUS OF CURVATURE AT THE AVERAGE LATITUDE BETWEEN TWO POINTS

UNLIKE ON A SPHERE, LENGTH OF A LINE FOR A GIVEN CHANGE IN LATITUDE AND LONGITUDE FROM ONE END TO THE OTHER DEPENDS NOT ONLY ON THE AZIMUTH OF LINE BUT ALSO ON HOW FAR NORTH (OR SOUTH) OF THE EQUATOR THE LINE IS

THE DIFFERENCE BETWEEN ARC AND CHORD LENGTHS IS ALSO INSIGNIFICANT FOR MOST SURVEY LINES

LS443 - INTRODUCTION TO GEODESY

Assignment No. 1

1. Compute **M**, **N** and **R** (Gaussian Mean Radius), on the **GRS80** ellipsoid, for a point whose coordinates are

$$\varphi = 46^{\circ} 06' 00.0'' \text{ N}$$

$$\lambda = 88^{\circ} 32' 00.0'' \text{ W}$$

The values should be expressed in **meters**.

2. A GLO township has been surveyed with its south line approximately coinciding with the Parallel of latitude at the latitude given above and the east line coinciding with the longitude given above. The average elevation within the township is 800 feet. South, east and west lines are exactly 6 miles each in length on the topographic surface.

- (a) Compute the latitude of the north line of township. Note that the east and west township lines run along meridian arcs at 800 feet elevation. For the purpose of this calculation, assume each of the arcs is an arc of a circle with radius equal to the meridian radius of curvature (**M**) at the latitude given above **PLUS** the elevation.
- (b) Compute the longitude difference between the east and west township lines. Note that the south and north township lines are parallels of latitude, and therefore, appropriate radii of curvature need to be used.
- (c) Compute also the length of the north line **in miles**, and the difference in length between the south and north lines, in **US survey feet**. This length difference between north and south lines, computed above, should also be checked by the following formula given on page 55 of BLM Manual of Instructions - 1973.

$$dm_{\lambda} = \frac{m_{\lambda} m_{\varphi}}{a} \tan \varphi \sqrt{(1-e^2 \sin^2 \varphi)}$$

where φ = mean latitude between the south and north lines

m_{λ} = length of south line along parallel

m_{φ} = length of east(or west) line along meridian

Note: You may disregard the geoid-ellipsoid difference for elevations. Use parameters of the **GRS80** ellipsoid and conversion factor for U.S. survey foot found on page 12 of State Plane Coordinate Manual (Appendix 2 of Class Notes). Use sufficient number of significant digits in computation to maintain precision to the nearest foot in all calculations.

All work must be clearly shown and submitted on 8 1/2 x 11" engineering or white paper. All sheets must be stapled together.

DIRECTIONS ON THE ELLIPSOID

- AZIMUTH OF LINES
- CONVERGENCE OF MERIDIANS
- FORWARD AND BACK AZIMUTHS

A SURVEY LINE HAS A DIFFERENT AZIMUTH AT EVERY POINT, SINCE AZIMUTH IS DEFINED AS THE CLOCKWISE ANGLE FROM NORTH

NORTH AT ANY POINT IS DEFINED BY THE MERIDIAN PASSING THROUGH THAT POINT, AND ON THE ELLIPSOID (AND ON EARTH) MERIDIANS ARE NOT PARALLEL

ABOVE ALSO LEADS TO THE FACT THAT THE DIFFERENCE BETWEEN FORWARD AND BACK AZIMUTH OF A LINE IS NOT EXACTLY 180° UNLIKE IN PLANE SURVEYING

NOTE THAT THIS DIFFERENCE IS DUE TO CONVERGENCE OF MERIDIANS AND NOT DUE TO TWO DIFFERENT NORMAL SECTIONS WHICH IS INSIGNIFICANT

THIS CHANGE IN AZIMUTH DUE TO CONVERGENCE OF MERIDIANS IS PROMINENT IN HIGH LATITUDES AND IS ABOUT 53" PER MILE IN EAST-WEST DIRECTION IN LATITUDE 45°

DEFLECTION OF VERTICAL, ASTRONOMIC AZIMUTH, AND LAPLACE CORRECTION

VERTICAL AT ANY POINT ON THE EARTH IS
THE DIRECTION OF GRAVITY VECTOR AT
THAT POINT

IN MOST PLACES ON EARTH, VERTICAL AND
NORMAL ARE NOT COINCIDENT OR
PARALLEL, AND THIS ANGULAR DEVIATION OF
THE VERTICAL FROM THE NORMAL IS CALLED
DEFLECTION OF VERTICAL

THIS DIFFERENCE IN NORMAL AND VERTICAL
ALSO LEADS TO TWO SETS OF COORDINATES
AND DIRECTIONS ON EARTH

LATITUDES AND LONGITUDES REFERENCED TO THE VERTICAL AT A POINT ARE TERMED ASTRONOMIC COORDINATES

TWO SETS OF MERIDIANS CREATED BY THE DEFLECTION OF VERTICAL NOT ONLY CHANGES THE AZIMUTH OF A LINE AT A POINT BUT ALSO CHANGES THE LONGITUDE OF THE POINT

FOR ALL GEODETIC COMPUTATIONS, DIRECTIONS ARE NEEDED TO BE REFERENCED TO THE NORMAL

DEFLECTION OF VERTICAL CREATES A PROBLEM AS ALL TERRESTRIAL MEASUREMENTS USING CONVENTIONAL INSTRUMENTS ARE DONE UNDER THE INFLUENCE OF GRAVITY FIELD

ALL DIRECTIONS (OR ANGLES) MEASURED BY CONVENTIONAL MEASUREMENTS ARE REFERENCED TO THE VERTICAL, AND SO ARE THE AZIMUTHS DIRECTLY MEASURED BY ASTRONOMICAL OBSERVATIONS

OBSERVED ASTRONOMIC AZIMUTHS CAN BE CONVERTED TO GEODETIC AZIMUTHS BY APPLYING A CORRECTION CALLED LAPLACE CORRECTION

REDUCTION OF MEASURED DISTANCES TO DATUM SURFACE

- HORIZONTAL DISTANCE
- DATUM DISTANCE
- MARK TO MARK DISTANCE

IN ALL PLANE SURVEYS, MEASURED SLOPE DISTANCES NEED TO BE CONVERTED TO HORIZONTAL DISTANCES BEFORE THEY ARE USED IN THE COMPUTATION OF COORDINATES

IN GEODETIC SURVEYS AND FOR COMPUTATION OF STATE PLANE COORDINATES, MEASURED DISTANCES NEED TO BE REDUCED TO THE DATUM (ELLIPSOIDAL) SURFACE AND THE LENGTH ON THE DATUM IS CALLED GEODETIC LENGTH

HORIZONTAL DISTANCE IN GEODETIC SURVEYING, UNLIKE IN PLANE SURVEYING, IS NOT A STRAIGHT LINE BUT A CURVED LINE (ARC)

FOR MOST SHORT LINES IN THE ORDER OF 10 MILES OR LESS, THE RIGHT ANGLE GEOMETRY USED FOR REDUCTION OF SLOPE LINE IN PLANE SURVEYING OFFERS SUFFICIENT ACCURACY

A SUITABLE RADIUS SUCH AS GAUSSIAN MEAN RADIUS SHOULD BE USED FOR REDUCING TOPOGRAPHIC HORIZONTAL DISTANCE TO DATUM DISTANCE

THIS REDUCTION USUALLY ASSUMES SHPERICAL ARCS

COMPUTATION OF GEODETIC COORDINATES FROM DISTANCE AND AZIMUTH OF A LINE

- DIRECT COMPUTATION
- INVERSE COMPUTATION

COMPUTATION OF COORDINATES OF POINTS
IS THE OBJECTIVE OF ANY SURVEY

IN PLANE SURVEYING, THIS IS DONE BY
APPLYING PLANE TRIGONOMETRIC
RELATIONSHIPS TO COMPUTE PLANE
COORDINATES

IN GEODETIC SURVEYING WHERE LINES ARE
CURVED AND COORDINATES ARE
CURVILINEAR, A DIFFERENT APPROACH HAS
TO BE USED

MOST FORMULAS THAT HAVE BEEN DEVELOPED MAKE CERTAIN ASSUMPTIONS, AND THEREFORE, OFFER VARYING ACCURACY OF COMPUTATIONS FOR LINES OF DIFFERENT LENGTHS

THE ASSUMPTIONS ARE NECESSARY DUE TO THE NATURE OF DATUM SURFACE AND THE FORMULA THAT IS USED DEPENDS ON THE LENGTHS OF LINES IN THE SURVEY AND THE REQUIRED ACCURACY

IN SPACE BASED TECHNIQUES SUCH AS GPS, COORDINATES OF POINTS ARE COMPUTED FIRST AND THE LENGTHS OF LINES ARE DERIVED FROM COORDINATES

MARK TO MARK, TOPOGRAPHIC DISTANCES ARE SOMETIMES COMPUTED FOR RECORD KEEPING PURPOSES

LS443 - INTRODUCTION TO GEODESY

Assignment # 2

1. Following data is available for a control point A, on NAD83 datum.

$$\begin{array}{lll} \phi = 45^{\circ} 58' 03.271'' \text{ N} & \lambda = 84^{\circ} 32' 42.563'' \text{ W} & \\ \text{Geoid Undulation} = 27.3 \text{ m.} & \xi = 2.34'' & \eta = -10.56'' \end{array}$$

The geodetic azimuth of a baseline from A to a second point B and the slope length AB were found to be $132^{\circ} 23' 43.1''$ and 17,875.224 feet, respectively by GPS measurements. Compute the

- (a) geodetic length of line AB, on NAD83 datum. The MSL elevations of the end points were 876 feet and 1343 feet.
- (b) latitude and longitude of point B
- (c) azimuth of line BA (back azimuth) by applying correction due to convergence of meridians (You need to compute correction by hand and apply it).
- (d) the expected azimuth of line AB if it were measured by star shots.

Note: For part (b), you may use the program **FORWARD** available in the diskette sent to you. You may also check your answer to part (c) from the results of this program.

2. (a) Compute the chord distance between points A and B on the ellipsoid from rectangular Cartesian coordinates (Equation. 2.3 in class notes). Compare and comment this distance with that computed in 1 above.

- (b) Use the **INVERSE** program to compute the ellipsoidal surface distance, forward and back azimuths of line AB. Compare with corresponding values obtained previously and comment. Express any change as a proportional error.

Note: Use sufficient number of significant digits in calculations, and attach printouts of output, if software is used. Complete work, in a logical order, should be presented on standard 8 1/2x11" sheets, stapled together. All answers obtained from computer software should also be shown along with the rest of the work. Merely Referring to computer printout for answers is NOT acceptable. Points will be deducted for illegible writing.

STATE PLANE COORDINATE SYSTEMS (SPCS)

WHY PLANE COORDINATES ?

- COMPUTATIONS INVOLVING PLANE COORDINATES ARE SIMPLER COMPARED WITH GEODETIC CALCULATIONS
- PLATS (AND MAPS) ARE MADE ON FLAT MEDIUM, AND GENERALLY, PLANE COORDINATES ARE USED TO PLOT THEM

PROBLEMS

- DISTORTIONS IN ANGLES, DISTANCES AND AREAS
- NEED A MAP PROJECTION FOR ACCURATE COMPUTATIONS

WHY STATE PLANE COORDINATES ?

- BASED ON MATHEMATICALLY RIGOROUS MAP PROJECTIONS
- DISTORTIONS ARE KNOWN (OR CAN BE COMPUTED)
- CAN COVER LARGE AREAS
- UNIFIED COORDINATE SYSTEM BASED ON NATIONAL DATUM
- NEEDED PARAMETERS ARE ALREADY AVAILABLE
- LEGALLY RECOGNIZED

MAP PROJECTION

- AN ORDERLY ARRANGMENT OF TWO SETS OF LINES OR CURVES THAT CONSTITUTE THE FRAMEWORK OF A MAP OR A CHART (GRATICULE)
- ON EARTH, TWO SETS OF CURVES REPRESENT MERIDIANS AND PARALLELS

MAP PROJECTIONS USED IN SPCS

- MATHEMATICAL FUNCTION(S) WHICH TRANSFORM GEODETIC COORDINATES, DISTANCES AND DIRECTIONS TO CORRESPONDING QUANTITIES ON A PLANE (GRID)

THREE METHODS OF GENERATING MAP PROJECTIONS, IN GENERAL

- GEOMETRIC (PERSPECTIVE)
- SEMI-GEOMETRIC
- MATHEMATICAL

PROJECTION IS FROM THE DATUM SURFACE TO A PLANE OR A DEVELOPABLE SURFACE (A SURFACE WHICH CAN BE DEVELOPED INTO A PLANE)

THREE CLASSES OF PROJECTIONS BASED ON GEOMETRY OF DEVELOPABLE SURFACE

- AZIMUTHAL
- CONICAL
- CYLINDRICAL

THE PROJECTIONS ARE ALSO CLASSIFIED ACCORDING TO THE POSITIONING OF THE DEVELOPABLE SURFACE ON DATUM SURFACE (ELLIPSOID)

- TANGENT
- SECANT
- POLY-SUPERFICIAL

ANOTHER WAY TO CLASSIFY PROJECTIONS IS BY USING THE ORIENTATION OF THE DEVELOPABLE SURFACE (PLANE) OR THE AXIS (CONE AND CYLINDER) WITH THE ROTATIONAL (MINOR) AXIS OF THE ELLIPSOID

- NORMAL
- TRANSVERSE
- OBLIQUE

EVERY MAP PROJECTION INTRODUCES DISTORTIONS (CHANGES) IN DISTANCES, DIRECTIONS (ANGLES) AND AREAS ON THE DATUM SURFACE I.E. THESE VALUES GET ALTERED IN THE PROCESS OF PROJECTING FROM DATUM TO PROJECTION SURFACE

SOME DISTORTIONS CAN BE ELIMINATED OR CONTROLLED MATHEMATICALLY

THREE CLASSES PROJECTIONS BASED ON DISTORTION CHARACTERISTICS

- EQUIDISTANT
- EQUAL AREA
- CONFORMAL

ALL THREE PROPERTIES CAN BE CONTROLLED BY CONTROLLING THE SCALE FACTOR AS DEFINED BELOW

$$\text{SCALE FACTOR} = \frac{\text{PROJECTED (GRID) LENGTH}}{\text{DATUM LENGTH}}$$

SCALE FACTOR IS A RATIO OF A LENGTH ON PROJECTION SURFACE TO THE CORRESPONDING LENGTH ON THE DATUM SURFACE AND CAN BE LARGER OR SMALLER THAN 1

SCALE DISTORTION OR SCALE ERROR IS THE AMOUNT OF DISTORTION INTRODUCED INTO THE DISTANCES, I.E. AMOUNT OF DEVIATION FROM A SCALE FACTOR OF UNITY

$$\text{I.E. S. E.} = |1 - \text{S.F.}|$$

THE SCALE ERROR OF A PROJECTION INCREASES AWAY FROM THE CIRCLE OF TANGENCY OR INTERSECTION OF THE DEVELOPABLE SURFACE WITH THE DATUM SURFACE

ELEVATION FACTOR IS THE CHANGE IN GROUND HORIZONTAL DISTANCE WHEN REDUCED TO DATUM SURFACE, I.E.

$$\text{ELEVATION FACTOR} = \frac{\text{DATUM LENGTH}}{\text{GROUND HOR. LENGTH}}$$

THE ELEVATION FACTOR DEPENDS SOLELY ON THE ELEVATION (FROM THE DATUM SURFACE) OF THE TERRAIN IN THE PROJECT AREA

ELEVATION FACTOR IS ALWAYS LESS THAN ONE AS THE TOPOGRAPHIC SURFACE IS ABOVE DATUM

IN AREAS WHERE THE SCALE FACTOR IS GREATER THAN ONE, ELEVATION FACTOR REDUCES THE SCALE ERROR AND WHEN THE SCALE FACTOR IS SMALLER THAN ONE, IT INCREASES THE SCALE ERROR

A COMBINED FACTOR IS THE PRODUCT OF SCALE FACTOR AND ELEVATION FACTOR

FOLLOWING ARE THE CONDITIONS NEEDED
FOR CONTROLLING THE DISTORTION
PROPERTIES OF A MAP PROJECTION

EQUIDISTANT ==> S.F. = 1

EQUAL AREA ==> PRODUCT OF S.F. IN ANY
TWO DIRECTIONS
PERPENDICULAR TO EACH
OTHER = 1

CONFORMAL ==> S.F. IN ANY TWO
DIRECTIONS
PERPENDICULAR TO EACH
OTHER ARE EQUAL

CONFORMALITY IS IMPORTANT IN SURVEYING BECAUSE

- PRESERVES ANGULAR RELATIONSHIP (BETWEEN SHORT LINES), AND THEREFORE, PRESERVES SHAPES OF FEATURES OF LIMITED SIZE
- SCALE FACTOR AT A POINT IS SAME IN EVERY DIRECTION, I.E. INDEPENDENT OF AZIMUTH OF LINE

CONFORMAL PROJECTIONS RESULT IN CHANGE OF

- DISTANCES
- AZIMUTHS

ALL CONFORMAL PROJECTIONS HAVE A SCALE FACTOR DIFFERENT THAN ONE, IN GENERAL, AND IS NOT CONSTANT OVER THE PROJECTION

EVEN THOUGH CONFORMAL PROJECTIONS PRESERVE THE ANGLES, THE DIRECTIONS OF LINES ARE CHANGED IN THE PROCESS OF PROJETING THEM

THIS IS DUE TO THE FACT THAT GEODETIC MERIDIANS ARE CURVED AND CONVERGE TOWARD THE POLE WHEREAS GRID MERIDIANS ARE A SET OF PARALLEL, STRAIGHT LINES

THIS ANGULAR DIFFERENCE BETWEEN TWO SETS OF MERIDIANS, CALLED CONVERGENCE ANGLE OR MAPPING ANGLE, AFFECT AZIMUTHS OF LINES

NOTE THAT, ON A CONFORMAL PROJECTION, ANGLES BETWEEN INTERSECTING LINES ARE UNCHANGED EVEN THOUGH AZIMUTH OF LINES MAY CHANGE

PROJECTIONS USED IN STATE PLANE COORDINATE SYSTEMS (SPCS)

- CONFORMAL
- USES THE REFERENCE ELLIPSOID AS DATUM
- AREA OF COVERAGE (ZONE) IS CHOSEN TO KEEP LENGTH DISTORTIONS WITHIN LIMITS ACCEPTABLE TO MOST SURVEYORS I.E. SCALE ERROR IS LIMITED TO 1:10,000
- SPCS-27 USED NAD-27 DATUM AND LINEAR UNIT WAS FOOT
- SPCS-83 WAS COMPUTED ON NAD-83 DATUM AND USES METER AS THE LINEAR UNIT
- MOST STATES HAVE ENACTED LEGISLATION TO LEGALIZE THE USE OF SPCS AND INDICATES WHICH LINEAR UNITS TO BE USED, I.E. A CONVERSION FROM ORIGINAL COORDINATES IS NECESSARY

WHICH STATES USE WHICH PROJECTIONS ?

THREE CONFORMAL PROJECTIONS, NAMELY

- LAMBERT CONFORMAL CONIC
- TRANSVERSE MERCATOR
- OBLIQUE MERCATOR

HAVE BEEN USED IN DESIGNING THE SPCS BY
THE NATIONAL GEODETIC SURVEY

OBLIQUE MERCATOR PROJECTION WAS USED
FOR ONE ZONE IN ALASKA AND ONE OF THE
OTHER TWO PROJECTIONS WAS USED FOR
REST OF THE STATES

THE OBJECTIVE IN SELECTING A PROJECTION IS TO REDUCE THE NUMBER OF ZONES NECESSARY TO COVER A STATE WHILE KEEPING THE SCALE ERROR TO 1:10,000 LIMIT LAMBERT CONFORMAL PROJECTION IS A CONIC PROJECTION, AND IS USED IN SPCS WITH A NORMAL ORIENTATION AND SECANT TO THE ELLIPSOID

THE CIRCLES OF INTERSECTION WITH THE ELLIPSOID ARE PARALLELS OF LATITUDE WHICH ARE CALLED STANDARD PARALLELS ALONG WHICH THE SCALE FACTOR IS UNITY

TRANSVERSE MERCATOR, AS IT IMPLIES, HAS THE AXIS OF THE CYLINDER AT A RIGHT ANGLE TO THE MINOR AXIS OF ELLIPSOID AND CYLINDER INTERSECTS THE ELLIPSOID ALONG TWO ELLIPSES WHICH ARE PERPENDICULAR TO THE EQUATOR

SCALE FACTOR ALONG A LINE MIDWAY BETWEEN INTERSECTING CIRCLES (OR ELLIPSES) IS THE SMALLEST AND THE INTERSECTING CIRCLES ARE CHOSEN IN A SUCH A WAY THAT THE SCALE ERROR AT THIS MIDWAY LOCATION IS WITHIN THE ALLOWED LIMIT

OUTER LIMITS OF THE PROJECTION ZONE ARE ALSO CHOSEN TO KEEP THE SCALE ERROR WITHIN ABOUT 1:10,000 AND THIS LIMITS THE WIDTH OF A ZONE TO APPROXIMATELY 156 MILES

THERE IS NO SCALE CHANGE IN A DIRECTION PARALLEL TO INTERSECTING CIRCLES, AND THEREFORE, THERE IS NO NEED TO LIMIT THE LENGTH OF A ZONE IN THAT DIRECTION

ABOVE ARE HELPFUL IS DECIDING WHICH PROJECTION IS TO BE USED FOR WHICH STATE AND HOW THE ZONES SHOULD BE DEMARCATED

LAMBERT PROJECTION IS SUITABLE FOR ANY STATE (OR A ZONE) WHICH HAS A LARGER EAST-WEST EXTENT AND T.M. PROJECTION IS SUITABLE FOR THOSE WHICH HAVE LARGER NORTH-SOUTH EXTENT

AS A PRACTICAL MATTER, ZONE BOUNDARIES ARE CHOSEN TO COINCIDE WITH ADMINISTRATIVE BOUNDARIES SUCH AS COUNTY BOUNDARIES

PROJECTION IS CENTERED ON A CHOSEN MERIDIAN, CALLED CENTRAL MERIDIAN, AND THIS IS USUALLY AT THE CENTER OF THE EAST-WEST SPAN OF THE ZONE

ORIGIN OF THE COORDINATE SYSTEM IS SHIFTED FROM ITS TRUE ORIGIN IN ORDER TO KEEP ALL COORDINATES POSITIVE

DEFINING CONSTANTS OF PROJECTIONS FOR EACH STATE/ZONE ARE FOUND IN *APPENDIX A OF THE STATE PLANE COORDINATE MANUAL* ATTACHED AS APPENDIX 2 OF PRINTED CLASS NOTES

COMPUTATIONS INVOLVED IN SPCS ARE THE FOLLOWING

- PLANE COORDINATES FROM GEODETIC COORDINATES
- GEODETIC COORDINATES FROM PLANE COORDINATES
- GRID SCALE FACTORS NECESSARY TO CONVERT GEODETIC DISTANCES TO GRID OR VICE VERSA
- CONVERGENCE ANGLES AND ARC-TO-CHORD CORRECTIONS NECESSARY TO CONVERT GEODETIC AZIMUTH TO GRID AZIMUTH AND VICE VERSA

COMPUTATION METHODOLOGY

- MANUAL USING
 - PROJECTION EQUATIONS
 - POLYNOMIAL COEFFICIENTS OR PROJECTION TABLES (LAMBERT CONFORMAL CONIC ONLY)
- AUTOMATED

MOST SOFTWARE DESIGNED FOR COGO CALCULATIONS AND/OR LEAST SQUARES ADJUSTMENTS HAVE THE ABILITY TO GENERATE SPCS FOR ALL STATES/ZONES

LS443 -INTRODUCTION TO GEODESY

Assignment # 3

1. The average elevation in certain project area is 1100 feet. There is a plane coordinate system covering the project area that will be used for all computations. If the surveyors want to use ground horizontal distances instead of grid distances what would be the maximum and minimum scale factors that would keep the scale error under 1/10,000. Note that the radius of earth needed for the calculation of elevation factor can be approximated as explained in chapter 4 of State Plane Coordinate manual (Appendix 2 of notes)
2. Following are the horizontal distance measured in a traverse. Find the grid length of these lines.

Line	Distance(feet)
AB	2781.89
BC	1947.43
CD	3954.28

Compute the grid distance for the two zones given below. Use the program **SPCS83** available in the disk to find the grid scale factor.

State	Michigan	Wyoming(Zone no.4902)
Ave. Latitude	46° 18' 20"	43° 03' 40"
Ave. Longitude	88° 11' 30"	107° 54' 10"
Ave. Elevation	800 feet	4380 feet

3. The geodetic coordinates, as determined by a GPS survey on NAD83, and other relevant data, are as follows:

	Point A	Point B
Latitude	47° 06' 39.068"	47° 06' 48.835"
Longitude	88° 32' 51.087"	88° 31' 28.046"
Geodetic Height(m.)	224.95	152.40

1. Compute State Plane coordinates, grid scale factors at points A and B. You may use the program **SPCS83** for this calculation only.
2. Compute the grid length and geodetic (ellipsoidal) length between A and B. Check your geodetic length by running **INVERSE** program and comment.

Note: Show all formulas and calculations. Pay attention to **significant digits** in the final answers. All other requirements are same as those for Assignment #2.

REDUCTION OF GROUND HORIZONTAL DISTANCES TO THE GRID

ELEVATION FACTOR

- GEODETIC HEIGHT
- ORTHOMETRIC HEIGHT
- GEOID HEIGHT

IF THE PROJECT AREA IS NOT VERY LARGE OR RUGGED, ONE ELEVATION FACTOR CAN BE USED FOR THE ENTIRE PROJECT

ELEVATION FACTOR SHOULD BE CALCULATED USING GEODETIC HEIGHTS (ELLIPSOIDAL HEIGHTS) NOT ORTHOMETRIC HEIGHTS (BENCHMARK ELEVATIONS)

ELLIPSOIDAL HEIGHTS ARE DIRECTLY COMPUTED FROM GPS OBSERVATIONS, OR CAN BE COMPUTED FROM ORTHOMETRIC HEIGHTS BY ADDING GEOID HEIGHT (GEOID UNDULATION)

THE AVERAGE GEOID UNDULATION IN THE CONTIGUOUS U.S. IS ABOUT 30 M. (100 FEET) AND IS NEGATIVE

GRID SCALE FACTOR

- CHANGES FROM POINT TO POINT
- SAME IN EVERY DIRECTION AT A POINT
- AVERAGE VALUE CAN BE USED FOR THE PROJECT

EVEN THOUGH THE SCALE FACTOR IS DIFFERENT AT DIFFERENT POINTS, BUT FOR MOST PRACTICAL APPLICATIONS S.F. COMPUTED AT A POINT IN THE MIDDLE OF THE PROJECT AREA WILL SUFFICE FOR MORE PRECISE WORK, SCALE FACTOR FOR EACH LINE NEED TO BE COMPUTED SEPARATELY AND IF THE LINES ARE VERY LONG THE AVERAGE OF S.F. AT TWO ENDS OF THE LINE MAY BE USED

IN EXTREME SITUATIONS, AVERAGE S.F.(k_{12}) FOR A LINE MAY BE COMPUTED AS FOLLOWS

$$k_{12} = k_1 + 4k_m + k_2$$

WHERE k_m = S.F. AT THE MID POINT OF LINE

REDUCTION OF MEASURED AZUMUTHS TO THE GRID

- ASTRONOMIC AZIMUTH
- GEODETIC AZIMUTH
- GRID AZIMUTH

IF ORIENTATION OF THE SURVEY IS BY ASTRONOMIC AZIMUTH OBSERVATIONS, THEY NEED TO BE FIRST CONVERTED TO GEODETIC AZIMUTHS BY APPLYING LAPLACE CORRECTION

DEFLECTION OF VERTICAL VALUES NEEDED FOR COMPUTING LAPLACE CORRECTION ARE AVAILABLE FROM NGS OR CAN BE GENERATED USING SOFTWARE *DEFLECT* AVAIALBLE FROM NGS (VISIT NGS WEB SITE www.ngs.noaa.gov)

DIFFERENCE BETWEEN GEODETIC AZIMUTH AND GRID AZIMUTH IS DUE TO

- DIFFERENCE OF GEODETIC AND GRID MERIDIANS (CONVERGENCE ANGLE)
- CURVATURE OF THE PROJECTED SURVEY LINE (ARC TO CHORD CORRECTION)

GRID AZ. = GEOD. AZ. - CONVERG. ANGLE +
ARC TO CHORD CORRECTION

$$t = \alpha - \gamma + \delta$$

CONVERGENCE ANGLE

- CAN BE POSITIVE OR NEGATIVE
- ZERO AT CENTRAL MERIDIAN AND INCREASES IN MAGNITUDE AWAY FROM C.M.

IN LAMBERT CONIC PROJECTION, PROJECTED GEODETIC MERIDIANS (NOT GRID MERIDIANS) TOO ARE STRAIGHT LINES, AND THEREFORE, THE SIZE OF THE CONVERGENCE ANGLE ONLY DEPENDS OF THE LONGITUDE CHANGE FROM THE CENTRAL MERIDIAN TO THE POINT IN QUESTION

BUT, IN T.M. PROJECTION, PROJECTED GEODETIC MERIDIANS ARE CURVES, AND THE SIZE OF THE CONVERGENCE ANGLE DEPENDS NOT ONLY ON THE LONGITUDE CHANGE BUT ALSO ON THE LATITUDE OF THE POINT

PROJECTED SURVEY LINES TOO ARE CURVED ON BOTH LAMBERT AND T.M. PROJECTIONS, AND THIS REQUIRES ANOTHER CORRECTION, CALLED *ARC-TO-CHORD* ($t - T$) CORRECTION, WHICH IS THE ANGULAR DIFFERENCE BETWEEN THE PROJECTED SURVEY LINE (ARC) AND STRAIGHT GRID LINE (CHORD)

ARC TO CHORD CORRECTION IS USUALLY INSIGNIFICANT FOR SHORT LINES (E.G.LESS THAN 5 MILES)

MAGNITUDE OF ARC TO CHORD CORRECTION DEPENDS ON

- LENGTH OF LINE
- DISTANCE FROM CENTRAL AXIS OF PROJECTION

SIGN DEPENDS ON

- AZIMUTH OF LINE
- WHICH SIDE OF THE CENTRAL AXIS

AXIS OF THE PROJECTION IS THE CENTRAL PARALLEL (LAMBERT) OR CENTRAL MERIDIAN (T.M.) WHERE THE S.F. IS SMALLEST

LS443 -INTRODUCTION TO GEODESY

Assignment # 4

Following is a part of a traverse run to establish control for a subdivision survey. Note that the length of lines are same as given in assignment #3.

Point	Length(feet)	Clockwise Angle
A		47° 11' 52"
1	2781.89	258 18 44
2	1947.43	93 18 12
3	3954.28
.	
B		

The traverse was run from point A to point B, for which the geodetic coordinates and other relevant data on NAD83 are as follows:

	Point A	Point B
ϕ	46° 18' 20.068" N	46° 06' 48.835" N
λ	88° 32' 51.087 W	88° 31' 28.046" W
η	8.234"	15.528"
Geodetic Height(m.)	224.9	152.40

Notice that the data for **ONLY** first three lines are given. The angle at A was between an azimuth mark and point 1. The geodetic azimuth of the azimuth mark was given as 332° 59' 58.2". The closing azimuth for the line closing at B was determined by a Polaris shot at B. This azimuth for the line from B to the last point of the traverse was determined as 115° 54' 05.9 ' .

(continued next page)

1. Compute State Plane coordinates, grid scale factors and convergence angles at points A and B.
2. What would be the starting **grid azimuth** at A? Disregard arc-to-chord(t-T) correction
3. If the closing **grid azimuth** for the line from the last point to B, obtained from traverse computations, is $296^{\circ} 59' 48.8''$, what is the angular closure of this traverse?
4. Compute state plane coordinates of first three points of the traverse.
5. Compute the grid distance, forward and back **geodetic azimuths** between A and B. **Hint:** Start by computing grid azimuth between A and B. Apply both the convergence correction and arc-to-chord(t-T) correction at each end.
6. Check the values computed in 5 above, by running **INVERSE** program. List the values obtained by the program.

Note: Use the parameters of NAD83 ellipsoid wherever necessary. Show all formulas and calculations Use **SPCS83** program wherever necessary and attach **printouts**. All other requirements are same as those for previous assignments.

UNIVERSAL TRANSVERSE MERCATOR (UTM) PROJECTION

- DEVELOPED BY U.S. MILITARY FOR WORLD-WIDE COVERAGE
- INITIALLY USED CLARKE 1866 ELLIPSOID AS DATUM AND COVERED A LATITUDE SPAN OF 80° S TO 80° N
- CURRENTLY USES GRS80 IN NORTH AMERICA AND THE COVERAGE ONLY FROM 0° TO 80° N LATITUDE
- EACH ZONE COVERS A LONGITUDE SPAN OF 6° AND HENCE THERE ARE 60 ZONES COVERING THE ENTIRE GLOBE
- METER IS THE LINEAR UNIT
- NOT SUITABLE AS A SURVEY GRID AS SCALE CHANGE IS TOO LARGE IN MOST AREAS AND CANNOT BE DISREGARDED
- WIDELY USED AS A COORDINATE CONTROL FOR GIS DATA LAYERS AS EACH ZONE HAS A LARGER GEOGRAPHIC COVERAGE

SURVEY MEASUREMENTS AND THE GRAVITY

SINCE MOST SURVEY MEASUREMENTS ARE MADE UNDER THE INFLUENCE OF GRAVITY, THEY ARE AFFECTED BY IT

ALL ANGLE AND DIRECTION MEASUREMENTS ARE AFFECTED BY THE DEFLECTION OF VERTICAL I.E. INSTRUMENT AXIS NOT ALIGNED WITH THE NORMAL TO ELLIPSOID

GEOID UNDULATIONS AFFECT THE REDUCTION OF LONG LINES, MEASURED ON THE TOPOGRAPHIC SURFACE, TO THE SURFACE OF THE ELLIPSOID

GEOID UNDULATIONS ARE NEEDED IF ORTHOMETRIC HEIGHTS ARE TO BE DERIVED FROM GPS MEASUREMENTS

ACCURATE ORTHOMETRIC HEIGHTS CANNOT BE DETERMINED THROUGH SPIRIT LEVELING AS EQUIPOTENTIAL SURFACES ARE NOT PARALLEL

ACCURATE ORTHOMETRIC HEIGHTS ARE NEEDED IN THE FOLLOWING APPLICATIONS

- TOPOGRAPHIC MAPPING
- CONSTRUCTION OF WATER SYSTEMS
- MONITORING OF WATER SYSTEMS
- OTHER CONSTRUCTION
- SUBSIDENCE STUDIES
- CRUSTAL MOTION STUDIES
- GRAVIMETRY FOR PROSPECTING AND TECTONIC STUDIES
- NAVIGATION
- PROPERTY RELATIONSHIP TO WATER

GRAVITY FIELD OF THE EARTH

- COMBINATION OF *GRAVITATIONAL ATTRACTION* AND *CENTRIFUGAL ACCELERATION*
- THE COMBINED EFFECT IS CALLED *GRAVITY*
- ACTS IN THE DIRECTION OF *PLUMBLINE*
- GENERALLY INCREASES FROM EQUATOR TO POLES
- THERE ARE ALSO LOCAL VARIATIONS DUE TO CHANGE IN MASS DENSITY OF EARTH
- THE PRESENCE OF THE FORCE FIELD OF GRAVITY CREATES A *POTENTIAL*

THE MAGNITUDE OF GRAVITY IS THE GRADIENT OF POTENTIAL IN A CERTAIN DIRECTION

$$\text{I.E. } g = -dw/dh$$

WHERE g = GRAVITY

w = POTENTIAL

AND h = ELEVATION ALONG PLUMLINE

NEGATIVE SIGN IS DUE TO THE FACT THAT GRAVITY INCREASES WHEN ELEVATION DECREASES

A SURFACE ON WHICH THE POTENTIAL HAS A CONTANT VALUE IS CALLED AN *EQUI-POTENTIAL* SURFACE OR A LEVEL SURFACE (OF GRAVITY)

GEOID IS A PARTICULAR EQUI-POTENTIAL SURFACE CHOSEN TO COINCIDE APPROXIMATELY WITH THE MEAN OCEAN SURFACE

THE *PLUMBLINE*, I.E. DIRECTION OF GRAVITY, IS PERPENDICULAR TO THE SURFACE OF AN EQUI-POTENTIAL SURFACE AT EVERY POINT ON THIS SURFACE

THE EQUI-POTENTIAL SURFACES ARE UNDULATED AS A RESULT OF LOCAL VARIATIONS OF GRAVITY, AND THEREFORE, PLUMBLINES CHANGE DIRECTION (DEFLECTION OF VERTICAL)

EQUI-POTENTIAL SURFACES ALSO CONVERGE TOWARDS THE POLES , AND THEREFORE, PLUMBLINES ARE CURVED, THE RESULT OF SYSTEMATIC INCREASE OF GRAVITY

ORTHOMETRIC HEIGHT IS THE LINEAR DISTANCE ALONG THE PLUMBLINE FROM THE GEOID TO THE POINT IN QUESTION

TWO POINTS HAVING THE SAME ORTHOMETRIC HEIGHT MAY NOT BE ON THE SAME EQUI-POTENTIAL SURFACE

DYNAMIC HEIGHT, ON THE OTHER HAND, IS THE DIFFERENCE IN POTENTIAL, EXPRESSED IN SOME LINEAR UNITS, BETWEEN THE GEOID AND THE POINT IN QUESTION

ORTHOMETRIC CORRECTION IS APPLIED TO PRECISE ELEVATIONS DETERMINED BY SPIRIT LEVELING, IN ORDER TO OBTAIN ORTHOMETRIC HEIGHTS

GRAVIMETRIC DETERMINATION OF GEOID

THE SURFACE OF THE GEOID IS DETERMINED IN RELATION TO A *REFERENCE ELLIPSOID*

THIS IS DONE BY BOTH *GRAVIMETRIC* METHODS AND *ASTRO-GEODETIC* METHODS

IN THE RECENT YEARS *SATELLITE ALTIMETRY* AND OTHER SATELLITE METHODS HAVE HELPED ACHIEVE GOOD RESULTS

THE GRAVITY FIELD OF THE REFERENCE ELLIPSOID IS DEFINED TO HAVE ONLY SYSTEMATIC VARIATIONS WITH LATITUDE, AND IS CALLED THE *NORMAL GRAVITY*

THE DIFFERENCE BETWEEN THE NORMAL GRAVITY(COMPUTED) AND THE ACTUAL GRAVITY(MEASURED ON EARTH AND REDUCED TO GEOID) IS CALLED *GRAVITY ANOMALIES(ΔG)*

IN GRAVIMETRIC METHOD FOLLOWING FORMULAS CAN BE USED TO DETERMINE THE *GEOID UNDULATIONS(N)* AND *DEFLECTION OF VERTICAL COMPONENTS*

$$N = (R/4\pi\gamma) \int_{\sigma} \Delta g \cdot s(\psi) \cdot d\sigma$$

$$\begin{Bmatrix} \xi \\ \eta \end{Bmatrix} = (1/4\pi\gamma) \int_{\sigma} \Delta g \, ds(\psi)/d\psi \begin{Bmatrix} \cos \alpha \\ \sin \alpha \end{Bmatrix} d\sigma$$

WHERE

R = SPHERICAL RADIUS OF EARTH

γ = AN AVERAGE NORMAL GRAVITY]

$d\sigma$ = SURFACE ELEMENT OF GRAVITY

ANOMALY, Δg

ψ = SPHERICAL DISTANCE(ANGLE) FROM POINT WHERE N IS COMPUTED TO Δg

$s(\psi)$ = STOKES' FUNCTION

α = AZIMUTH OF Δg

IF SUFFICIENT GRAVITY ANOMALIES ARE AVAILABLE, ABOVE EQUATIONS CAN BE EVALUATED BY SUMMATION OF FINITE SURFACE ELEMENTS

ONE PROBLEM IN GETTING AN ACCURATE ESTIMATE OF THE GEOID IS NON-AVAILABILITY OF GRAVITY DATA FOR THE WHOLE EARTH

VERY ACCURATE VALUES CAN BE OBTAINED BY INTEGRATING OVER A LARGE AREA EG. $\psi = 100^\circ$ AS THE INFLUENCE OF GRAVITY ANOMALIES BEYOND THIS IS LESS THAN 1"

A COMBINATION OF *SPHERICAL HARMONICS* AND STOKES' FORMULA IS USUALLY EMPLOYED